A Minimum-Offer Contribution Mechanism for the Provision of Public Goods*

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August 25, 2023

Abstract

Public goods provision continues to be a major problem of interest for economics. Many current methods require government level intervention. In cases where governments are unable or unwilling to intervene, lower power mechanisms are required. Current best practice in this case is the provision point mechanism, typically used by Kickstarter, but this mechanism does not eliminate the potential for the free rider problem. We propose a novel Minimum-Offer Contribution Mechanism where each player makes an offer and then each contribution is equal to the lowest offer made. This mechanism eliminates the free rider problem and implements the Lindahl (1958) equilibrium in weakly dominant strategies.

1 Intro

A great deal of work in economics has been devoted to finding ways to efficiently provide public goods (and avoid public bads) in the presence of the free rider problem, but the problem persists in a number of major arenas. For example, international cooperation on environmental issues has not been as fruitful as is generally desired and current public goods

*Neither Nathaniel Neligh nor any immediate family member has any financial interests related to this research. The experiment was carried out under the approval of the UTK IRB. The study was pre-registered with the AEA RCT registry. Thanks to Matt Van Essen and the participants of the UTK seminar for feedback. Also, thanks to Nancy White for her proofreading and editing.

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mechanisms have not fixed the issue. In this paper we propose a novel mechanism which could be beneficial in some of these unresolved environments.

A large portion of the literature has been devoted to finding mechanisms which allow a center to provide an efficient amount of the public good in the presence of imperfect information.[1] However, the mechanisms involved require the ability to create and destroy money and/or the ability to enforce participation. This makes the mechanisms impractical for any environment where there is not a government or equivalent central authority to implement them. We refer to these mechanisms as low-information/high-power mechanisms, because they do not require information to work but do require a center with specific powers.

This paper looks at a different problem, how can we promote the provision of public goods when no powerful authority exists but information is present? International cooperation provides a relevant application here. The impacts of international public goods are often roughly proportional to the scale of different industries and economic activities within a country. For example, a percentage reduction in fishing caps would impose a cost that is roughly proportional to the size of a country's fishing industry. Therefore at least rough information is available, but overarching authority is absent.

In this paper, we propose a Minimum-Offer Contribution Mechanism (MOCM) where players make offers simultaneously and then pay out proportional to the lowest adjusted offer among all participants. This approach eliminates the free rider problem in a manner that has several appealing properties. The mechanism is budget balanced, individual rational and implements the strongly efficient outcome in weakly dominant strategies. No government type authority is needed to enforce participation or create money. Although some mechanism or entity will be needed to assign prices, they do not need any further power and any sufficiently disinterested party should be sufficient. Of course, something must be sacrificed for not needing a powerful implementer, so our mechanism has higher information requirements. We call such mechanisms high-information/low-power mechanisms, although we will show our mechanism can still be beneficial in low information environments with some adjustments.

The appealing properties of the MOCM come from three technical features of the mechanism. First, offers are adjusted based on Lindahl prices which allows for fair efficient outcomes.[2] Lindahl prices are hypothetical prices for public goods under which every individual

[1] Clarke (1971); Groves and Loeb (1975); Myerson and Satterthwait (1983); Laffont (1987); Falkinger et al. (2000); Grüner and Koiryama (2012)

[2] Lindahl (1958)

pays a price for the good that is equal to their marginal benefit and where every player demands the same amount of the public good. The use of Lindahl prices helps match individual marginal incentives with planner incentives to implement efficient outcomes. Second, the pinning of contributions to a single pivotal player allows us to match these marginal incentives while maintaining a balanced budget. This also means the mechanism generates defined off-path outcomes, which do not exist in the standard Lindahl framework. Third, using the minimum offer as our pivot ensures the mechanism is individually rational for all players and makes sure no one pays more than their offer. Note that the minimum offer element essentially provides agents with veto power, which is instrumental for ensuring that the mechanism provides Pareto improvements. This point was first argued by Wicksell (1958) and more recently discussed by Van Essen and Walker (2017). Because the MOCM implements the Lindahl (1958) equilibrium, the outcome will be fair in the sense that players pay in proportion to their marginal benefit from the public good at equilibrium. Lindahl (1958) equilibrium is generally considered a very fair equilibrium.[3]

Traditionally, the Wilson (1987) critique has been leveled at high-information mechanisms due to their reliance on a mechanism maker with knowledge of preferences. If preferences are known, why do the agents not simply contract on an efficient outcome? Unfortunately, in the real world the act of negotiating a contract is often characterized by barriers and frictions.[4] While low-information/high-power mechanisms like the VCG mechanism[5] have excellent theoretical properties, it is more often low power higher information mechanisms that see widespread use. For example, consider the provision point mechanism (PPM) proposed by Bagnoli and Lipman (1989). Their mechanism involves setting a contribution threshold equal to the efficient amount. If offers meet the threshold, the good is provided. Otherwise, it is not. The PPM mechanism is used frequently by charities[6] and on crowdfunding platforms like Kickstarter. Also, non-profits like National Public Radio or the Sierra Club regularly use matching donation mechanisms which are also in this category. As we show in Section 3, the matching contributions mechanism is a variant special case of our mechanism.

The PPM is the current predominant low-power high-information mechanism, so we will use it as a point of reference. The MOCM has several advantages over the PPM. First

[3] Sato (1987); Buchholz and Peters (2007)

[4] There is a large literature on the topic of frictional and incomplete contracts. A small sample of the related papers includes Antràs (2003); Antràs (2005); Acemoglu et al. (2007).

[5] Clarke (1971); Groves and Loeb (1975)

[6] Bagnoli and McKee (1991)

and foremost, in the MOCM the efficient outcome is implemented as the result of each player choosing their unique weakly dominant strategy. This is potentially an important contribution, since threshold-based mechanisms can lead to coordination problems where players do not contribute in the hopes of getting their more preferred equilibria (ones where they do not pay as much). Coordination issues are of primary concern in most unresolved high-information environments.

Both the PPM and the MOCM require information to perform optimally, but the information each requires is essentially orthogonal. Setting the correct provision point in the PPM requires knowing the efficient provision level but not how much each player benefits from the public good. On the other hand, setting the correct prices in the MOCM requires knowing what portion of the marginal public good benefits each player receives, but not what the efficient provision level is. Therefore, in settings where only one type of information is available, the best choice of mechanism will be straightforward.

While it is designed for high-information environments, the MOCM is somewhat robust to imperfect information and price manipulation. In Section 3.0.1 we discuss how well the MOCM performs in different imperfect price conditions. The MOCM outperforms the Voluntary Contribution Mechanism (VCM) in the presence of manipulation as long as the manipulator does not have a majority stake in the public good, and even then the MOCM may perform better in many cases. In cases where the participants have more information about each other's preferences than the price setter does, Appendices C and D, discuss how players can be induced to give up their information.

The biggest weakness of the MOCM is imperfect information and heterogeneous benefits combined with large groups. When there are enough potential contributors with enough heterogeneity, inevitably a player with near zero benefit from the public good will be assigned a high price leading to total contributions approaching zero. However, the MOCM can be modified to be helpful in such cases. Section 4.1 provides a version of the mechanism to be used by charitable organizations where the set of potential donors is large and not well known at the individual level. This N-Group MOCM can be thought of as a generalization of the common matching donation mechanism, which would be considered a 2-Group MOCM. The N-Group MOCM has the potential to generate more contributions than the 2-Group MOCM in many scenarios.

We contend that the MOCM is a potentially valuable addition to the public goods toolbox which can be particularly valuable for approaching scenarios with few agents, severe coordination problems, and no overarching authority. International cooperation on environmental issues seems to exhibit these features, so it presents a situation where our mechanism could be particularly useful. It is difficult for countries to hide information about their preferences since the size of major polluting industries is relatively easy to approximate with publicly available information. The N-Group version of the MOCM may also be useful in general fundraising activities.

1.1 Literature

The literature on public goods provision is too large to discuss here with any level of completeness, so we will only briefly mention what we consider to be the two most relevant papers.

One of the closest things in the literature to the MOCM was proposed as a charity auction mechanism by Goeree et al. (2005). They examined games where an item was auctioned off to raise money for a linear public good. Goeree et al. (2005) found that the minimum price all pay auction was the revenue maximizing auction. Our papers describe sets of models with a small overlap. If the benefit of the public good is linear and the impact is homogeneous in the MOCM, the resulting model is identical to a Goeree et al. (2005) minimum price auction where the value of the auctioned good is zero.

Empirically, a mechanism like the one proposed by Goeree et al. (2005) did not perform well when tested.[7] It failed to reach theoretical contribution levels and failed to beat other public good funding mechanisms. We hypothesize that the auction framing may have effectively reintroduced the free rider problem psychologically even while the minimum price component eliminated the free rider problem mathematically. Individuals are often reluctant to contribute to public goods, since they do not want to subsidize free riders. Similarly, individuals are likely to be reluctant to bid in all pay auctions, because they do not want to subsidize the single auction winner. In both cases, participants do not want to risk footing the bill for someone else's unfair/unearned benefit.

The other most similar element of the literature is the matching mechanism explored by Guttman (1986) where one player offers to match some fraction of the contributions of another player. The MOCM can be thought of as similar to a special case of matching contributions with an optimal matching fraction based on Lindahl prizes. There is still a difference however, in that the MOCM allows players to only match contributions up to a

[7] Corazzini et al. (2010)

certain amount. This upper limit prevents losses when other players over-contribute and allows for dominant strategy implementation of the efficient public goods provision level.

2 Basic Setup

We begin by using a specific functional form that makes the exposition clean and simple. We will generalize our main result later.

First we construct a basic public goods game. There are players $i \in \mathcal{I} = \{1, ..., I\}$. All players make offers $\boldsymbol{x} = (x_1, ..., x_i, ..., x_I)$ simultaneously where $x_i \in \mathbb{R}^+$. Based on the offers made, a set of contributions, $\boldsymbol{y} = (y_1, ..., y_i, ..., y_I)$, is generated by some mechanism. The mapping from offers to contributions depends on the mechanism involved.

Individuals get utility from the total public good provided and private consumption in the form

$$u_i(Y, y_i) = \delta_i g(Y) - y_i \tag{1}$$

Where $Y = \sum_{i=1}^{I} y_i$. Here $g(\bullet)$ is a weakly concave, differentiable, increasing function that represents the total benefit from the public good and δ_i is the share received by individual i. Assume that $\sum_{i=1}^{I} \delta_i = 1$ without loss of generality. We assume that $g(\bullet)$ satisfies the Inada-like conditions that $\frac{d}{dY} g(0) > \frac{1}{\delta_i} \forall i$ and $\lim_{Y \to \infty} \frac{d}{dY} u_i(Y, x_i) < I$ so public goods provision is always positive and finite in equilibrium.

Throughout the proofs and propositions in this paper, multiple equilibria are possible due to weak concavity, but if one assumes strict concavity of preferences this is no longer needed. Note we allow players to contribute more than their endowment. Any bounds on contributions are handled by assigning arbitrarily low utility values when contributions are too high.

In terms of solutions, we will be focusing primarily on equilibrium in weakly dominant strategies but we will also consider pure strategy Nash equilibria when weakly dominant strategies are not available.

2.1 Efficiency

Efficiency is the ultimate goal of public goods mechanisms. In this setup, a Pareto Efficient allocation is a set of contributions y_i^* summing to Y^* for which there is no other allocation which provides all players at least as high of payoffs and at least one player higher payoffs. Samuelson (1954) showed that an allocation in a public goods economy is Pareto Efficient, if and only if

$$\sum_{i=1}^{I} MRS_i(Y^*, y_i^*) = 1$$

Where MRS_i denotes player i's marginal rate of substitution. Given our setup this condition simplifies to

$$g'(Y^*) = 1$$

Because we are using quasi-linear utility, the set of Pareto efficient allocations is equal to the set of strongly efficient allocations, ie those which maximize the total sum of utility.

2.2 The VCM

As a point of reference, we first discuss the public goods contribution problem in its basic form. When the amount players offer is equal to the amount they contribute to the public good, that is the VCM. Under the VCM, each player is picking

$$y_i = \arg\max_{y_i} \delta_i g(Y) - y_i$$

With a total equilibrium contribution level $Y = \sum_{j=1}^{I} y_i$. Call an equilibrium total contribution level in the VCM Y_V^*. In our setup, this solution satisfies the condition

$$g'(Y_V^*) = \frac{1}{\max_i \delta_i}$$

Which means the Samuelson condition is generally not satisfied in this case and public goods are under-provided. In this environment, only players who are tied for maximum δ_i will contribute.

2.3 The MOCM

We now present the MOCM formally. In the MOCM, each player makes an offer x_i of how much they are willing to contribute to the public good. Each player is also given a weight w_i. The actual contributions for each player are given by

$$y_i = w_i * \min_j \left(\frac{x_j}{w_j}\right)$$

Where p_i is the price for player i and $\sum_{i=1}^{I} p_i = 1$. Total contributions are given by

$$Y = \left(\sum_{i=1}^{I} w_i\right) \min_j \left(\frac{x_j}{w_j}\right) = \min_j \frac{x_j}{w_j}$$

Under this mechanism, players pay out a weighted amount based on the lowest adjusted offer. In the case where p_i is identical across individuals, this corresponds with each player contributing the minimum offered amount.

In the MOCM mechanism each player i picks x_i to solve

$$x_i^* = \arg\max_{x_i} \delta_i g_i \left(\min\left(\frac{x_i}{w_i}, \frac{x_k}{w_k}\right)\right) - \min\left(x_i, w_i \frac{x_k}{w_k}\right)$$

where

$$k = \arg\min_{j \neq i} \frac{x_j}{w_j}$$

2.4 Lindahl Equilibrium

Next we introduce a concept which is important for understanding the result of the MOCM: Lindahl Equilibrium. A Lindahl equilibrium is a strongly efficient outcome that is is generally considered "fair" and is a gold standard for public goods outcomes. However, Lindahl Equilibrium is not implementable in a game theoretic sense as it is originally frame.

In public goods games, a Lindahl equilibrium is a set of prices, $\boldsymbol{p}^*$, and a level of total contributions, Y_L^*, such that each player demands that level of contribution given the price they pay and the budget is balanced.

In our setup a Lindahl Equilibrium is a set of contributions Y_i^* and a set of prices p_i^* such that

$$Y_L^* \in \arg\max_{Y_i} \delta_i g(Y_i) - p_i^* Y_i \forall i$$

$$y_i = p_i^* Y_i$$

Since there is no rule defining what happens outside of equilibrium when different players demand different quantities of the public good. This is one of the major issues the MOCM is designed to solve.

Foley (1970) has shown that under the conditions we assume, the Lindahl Equilibrium exists and is Pareto Efficient/Strongly Efficient. Efficiency can be seen from the optimality condition

$$MRS_i(Y^*, y_i^*) = p_i^*$$

and the fact that budget balance guarantees

$$\sum_{i=1}^{I} p_i^* = 1$$

Together these two facts give us Samuelson's Condition. This means $Y_L^* = Y^*$. In our environment, if we set $p_i = \delta_i$, then the player's Lindahl optimization problem becomes

$$\arg\max_{Y_i} \delta_i g(Y_i) - \delta_i Y_i = \arg\max_{Y_i} g(Y_i) - Y_i$$

Which is the planner's problem. All players demand Y_L^* and prices sum to 1 meaning that $p_i^* = \delta_i$ are the Lindahl Prices. It is intuitive that this pricing would yield the efficient outcome, because players are required to pay for the public good in proportion to the benefit they receive.

2.5 Properties of the MOCM

We begin with the result showing that the outcome of the MOCM matches with our standard for efficiency and fairness.

Theorem 1. *The MOCM with weights equal to Lindahl prices implements the corresponding Lindahl Equilibrium in weakly dominant strategies.*

Proof. Under the MOCM, the individual's offer selection problem can be rewritten as a problem where the individual selects a contribution level subject to a restriction imposed by the other offers.

$$Y_i^{M*} = \arg\max_{Y_i} \delta_i g\left(\min\left(Y_i, Y_{min}^i\right)\right) - w_i \min\left(Y_i, Y_{min}^i\right) \quad (2)$$

Where

$$Y^i_{min} = \min_{j \neq i} Y_j$$

Player i's objective function in the Lindahl Equilibrium is

$$Y_i^{L*} = \arg\max_{Y_i} \delta_i g\left(Y_i\right) - p_i^* Y_i$$

Given that $g(\bullet)$ is increasing and concave, the objective function in the MOCM is increasing in Y_i up until Y_i^{L*} if $w_i = p_i^*$. Therefore, regardless of Y^i_{min}, as long as $w_i = p_i^*$, it is weakly optimal to pick $Y_i^{M*} = Y_i^{L*}$. $\square$

For proof of the general version see Appendix A.1. Recall that in this case δ_i is the Lindahl price for player i and therefore it is also the optimal weight in the MOCM.

This is the most important feature of the mechanism. It eliminates the free rider problem by refunding those who make high offers relative to others. It is important to note that, while the MOCM does implement the efficient outcome as the result of each player choosing their unique weakly dominant action, there are still other equilibria. For example, there is another equilibrium where $Y_i = 0 \forall i$. However, these other equilibria are generally not efficient and are not the result of dominant strategies, so they are generally less plausible.

This result also guarantees a level of "fairness" in the outcome, because players pay into the public good relative to their equilibrium marginal benefit from the public good.

In addition to implementing the efficient outcome, this mechanism also has a number of nice features. First, it is budget balanced. The amount spent on the public good is the exact sum of contributions. This is very helpful for mechanisms that can't rely on governments or other entities which can destroy and create money.

Second, no player will ever pay out more than their offer even if the weights are wrong, because

$$w_i * \min_j \tfrac{x_j}{w_j} \leq x_i$$

As a result, the mechanism could be implemented through a system of refunds.

Third, the mechanism is individually rational in the sense that a rational player choosing Y_i optimally will always make weakly more than 0 regardless of how others play. This mechanism always generates Pareto improvements with optimizing agents due to the veto power held by individual contributors.

2.6 Generalized Utility

Note that the functional form is unimportant for this main result, although it will be used in later results. Neither Theorem 1 nor its proof depend on the quasi-linear utility function from expression 1. We can apply the same result without modifying the wording to any utility function of the general form

$$u_i(Y, \omega_i - y_i) \tag{3}$$

where $u_i(\bullet)$ is weakly concave and increasing. The ω_is represent each player's budget, although they are not strictly needed and can generally be ignored for purposes of optimization. We do not rule out negative wealth or players contributing more than their endowment. In this general setting, the MOCM also maintains its beneficial features. The mechanism is still "fair", individually rational, budget balanced, and never requires people to pay more than their offer.

The proportional benefit functional form used in most of the paper is used for two reasons. First, it makes it much easier to compare the outcomes of imperfect implementations of the MOCM. Second, it makes it easier to find the Lindahl prices. Lindahl prices are not, in general, easy to see from utility functions just by inspection.

3 Imperfect Weights

As mentioned, the baseline MOCM requires a mechanism maker who can set the correct $\boldsymbol{p}^*$. In many cases the mechanism maker and participants may not have common knowledge of all players' preferences. If the mechanism maker is aware of the correct $\boldsymbol{p}^*$ but the player's are not, this presents no issue due to the dominant strategy nature of the implementation.

If only the mechanism maker is unaware of the true $\boldsymbol{p}^*$, efficiency is still achievable with only minor sacrifices (see Appendix C). In cases where preferences are not common knowledge, it may be possible to extract a player's information about the incentives of others. However, this extraction may be incomplete, since information about the preferences of others will often contain information about one's own preferences. To see one example where information is extracted efficiently see Section D.

In the rest of this section, we focus on showing what exactly happens when the weights are not assigned correctly, either due to manipulation or lack of information.

We will want some easy way to compare outcomes of the MOCM to the VCM in the presence of manipulation and imperfect information. Note that we will not be theoretically comparing the performance of the MOCM and the provision point mechanism under imperfect information, because the distortions affecting these two mechanisms are essentially orthogonal. The MOCM is impacted by distortions in δ_i but not $g(Y)$ while the provision point mechanism is impacted by distortions in $g(Y)$ but not δ_i. The results are therefore largely mechanical and come down to determining which type of distortion is larger given assumptions.

Given the proportional benefit structure of this game, we can make use of the following proposition to easily compare outcomes.

Remark 1. If $Y(\kappa) = \arg\max_Y \kappa * g(Y) - Y$, then $Y(\kappa)$ is increasing in κ (in the strong set order sense).

This result is a straightforward application of Milgrom and Shannon (1994)'s comparative statics. The first best outcome(s) correspond to $\kappa = 1$ while the VCM outcome corresponds to $\kappa = \max_i \delta_i$.

3.0.1 Incorrect Prices

First we consider what happens when a mechanism is implemented imperfectly in a general sense. This will also be useful in examining the impact of other specific error types. Say that the mechanism assigns a potentially incorrect weight w_i to each participant.

In this case each participant is choosing

$$Y_{inc}^i \in \arg\max_{Y_i} \delta_i * g\left(\min\left(Y_i, Y_{min}^i\right)\right) - w_i * \min\left(Y_i, Y_{min}^i\right) \tag{4}$$

Where

$$Y_{min}^i = \min_{j \neq i} Y_j$$

Proposition 1. *The MOCM with incorrect prices can implement in dominant strategies the solution to*

$$Y_{inc}^* \in \arg\max_Y \left(\min_i \left(\frac{\delta_i}{w_i}\right) g(Y) - Y\right)$$

For proof see Appendix A.2. In this case the corresponding κ value which determines the efficiency of the outcome is $\min_i \left(\frac{\delta_i}{w_i}\right)$. The lower $\min_i \left(\frac{\delta_i}{w_i}\right)$, the worse the MOCM performs.

3.0.2 Manipulation

With the aid of Proposition 1 we can now consider what happens when an interested party with full information selects the prices. Say that one agent, j, assigns prices w_i for all players with the restriction that $\sum_{i=1}^{I} w_i = 1$. He knows the true p_i^*s, but he is a normal player, meaning he gains some benefit from the public good and is potentially on the hook for some of the cost. The game operates in two stages. First, the manipulator chooses w_is, then all players (including the manipulator) make their offers simultaneously. Assume that after weights are assigned, players play the equilibrium described in Proposition 1.

The result is given by the following proposition

Proposition 2. *If the mechanism weights are determined by individual j, the MOCM will implement Y_{man}^* total contributions in weakly dominant strategies, where*

$$Y_{man}^* = \arg\max_Y \left(\frac{1-\delta_j}{1-w_j} g(Y) - Y\right)$$

and $w_j \in [0, \delta_j]$

For proof see Appendix A.3. In this case, the corresponding k value is $\frac{1-\delta_j}{1-w_j}$. The exact optimal value for w_j depends on the structure of $g(Y)$ but it will always be weakly less than δ_j.

Then there are two potentially competing pulls on the manipulator. They want to maximize the minimum adjusted offer from other players but they also want to minimize their own price w_j.

By changing the values of w_i, they can reduce the fraction of the public good they have to pay for, but any manipulation will also reduce the maximum possible amount of public good provided, since manipulation will always reduce $\min_i \left(\frac{\delta_i}{w_i}\right)$. For a given w_j, the manipulator will want to equalize $\frac{\delta_i}{w_i}$ for all other players, so there is no incentive to distort the relative prices of the other players.

3.1 Comparison of Outcomes

We now compare the outcomes in the manipulation and imperfect information cases with the efficient and VCM outcomes. We can use Remark 1 for this comparison, since the discussed outcomes implement a total contribution that can be written in the relevant form. This allows rank various scenarios in terms of total public goods contributions based on their κ values. The following table summarizes the result

Mechanism/Situation	κ	Implemented Y
Social Planner	1	Y^*
MOCM Perfect Conditions	1	Y^*
VCM	$\max_i \delta_i$	Y_V^*
MOCM Incorrect Information	$\min_i \left(\frac{\delta_i}{p_i}\right)$	Y_{inc}^*
Homogeneous Price MOCM	$\min_i (I\delta_i)$	Y_{hom}^*
MOCM Manipulation	$\frac{1-\delta_j}{1-p_j}$	Y_{man}^*

The Homogeneous Price MOCM refers to an MOCM where $p_i = \frac{1}{I}\forall i$. We include this to as a potential baseline when marginal benefit information is not available or is ignored.

Note that in all cases $\kappa \leq 1$, so there is no risk of inefficient over-contribution. We know that Y^* is optimal and higher than the other implemented Ys in the table, but it is not immediately obvious how Y_V^*, Y_{inc}^*,Y_{hom}^*, and Y_{man}^* rank. With closer inspection we can see some general tendencies.

For example, the MOCM with manipulation will be more efficient than the VCM in many circumstances.

Corollary 1. *The VCM can only be more efficient than the MOCM with manipulation if the manipulator's* $\delta_j > 0.5$.

It is only possible for $\min_i \left(\frac{\delta_i}{p_i}\right)$ to be greater than $\frac{1-\delta_j}{1-p_j}$ if $p_j \geq 0.5$. It generally not common for public goods to provide more than half their benefit to a single individual. Note that $\delta_j > 1/2$ does not guarantee that the MOCM with manipulation performs worse than the VCM, but we can construct examples where it does. See Appendix B for such an example.

The Homogenous Price MOCM will outperform the VCM as long as the player that gets the least benefit from the public good gets at least $1/I$ of the benefit for the player with the highest benefit. Therefore, the Homogeneous Price MOCM should perform well in cases without extreme heterogeneity of benefits.

3.2 Random Noise and Imperfect Information

In general, it is only in the cases of high heterogeneity and low information that the MOCM loses out to the VCM. Under certain circumstances, larger groups can make the problems more acute.

Consider what happens when player i's benefit from the public good is determined by a random variable η_i so

$$u_i(Y, y_i) = \eta_i g(Y) - y_i \tag{5}$$

Say η_is are drawn independently from a distribution with a PDF $f_\eta(\bullet)$, maximum of $\bar{\eta}$, and a minimum of $\underline{\eta}$. Note that we do not require the restriction that η_is sum to one since that restriction makes no sense when comparing differing numbers of players. The true Lindahl price in this situation would be $p_i^* = \frac{\eta_i}{\sum_j \eta_j}$.

In this environment the κ for the VCM is $\max_i \eta_i$ which converges to $\bar{\eta}$ as the number of players increases.

If the center has no information, the best they can do is the homogeneous MOCM. The κ for the homogeneous MOCM is $\min_i I\eta_i$. We must consider several cases. If $\underline{\eta} \geq 0$ then $\min_i I\eta_i$ goes to infinity with I. This is trivially better than the VCM. If $\underline{\eta} = 0$ and $f_\eta(\bullet)$ is continuous then $\min_i I\eta_i$ is converges to an exponential distribution with mean $f_\eta(0)$. Whether this is better than the VCM depends on circumstance.

The real unambiguous problem for the MOCM arises when $\eta_i \leq 0$ with positive probability. In this case, total contributions inevitably go to zero as the population increases. For these situations, we developed the N-Group MOCM discussed in Section 4.1.

4 Generalization and Robustness

In the final major section of this paper we discuss generalization and robustness of the MOCM in the face of different utility functions and imperfect information.

4.1 The N-Group MOCM for Charitable Contributions

We now discuss an approach to dealing with high-heterogeneity, low information environments with many people, the N-Group MOCM. For the discussion of this mechanism we will use Nash Equilibrium as our solution mechanism, because the within group refund mechanism creates an off path coordination issue that removes the dominant strategy solve-ability of the game.

The N-Group MOCM

Say that there are N groups indexed $k \in \{1, 2, ..., N\}$. Group k contains the set of individuals with indexes $i \in S_k$.

Define the total group offer

$$X_k = \sum_{i \in S_k} x_i$$

The mechanism states that

$$y_i = x_i + \tfrac{1}{|S_k|} \left(X_k - \min_l X_l \right)$$

This means that each player gets an equal refund if their group offered more than the lowest total group offer. We could make the refund proportional to the players' offer, but this would not fundamentally change the results.

Note that under this mechanism

$$Y = \min_k X_k$$

In this setting it is useful to define the unbounded group solution.

$$\tilde{X}_k(M) = \sum_{i \in S_k} \arg\max_{y_i} u_i \left(M \sum_{j=1}^{I} y_j, -y_i \right)$$

In other words $\tilde{X}_k(M)$ is the total contributions value determined by group k if the contributions of other groups did not constrain them. In cases where the VCM equilibrium is not unique, a fixed arbitrary element is selected.

Proposition 3. *There exists a Nash Equilibrium of the N-Group MOCM game where* $Y = M \min_k \tilde{X}_k(M)$ *with each group contributing* $\min_k \tilde{X}_k(M)$*.*

For proof see Appendix. A.5 There are three things to consider from this result.

First, equilibrium is no longer in weakly dominant strategies. In the case of one group contributing more than another, there needs to be some way to deal with the excess contributions. Any method one can employ introduces some level in coordination problem. However, coordination is still less of an issue than in the PPM and VCM cases, because free riding only exists within groups. Between groups there is no free riding, since all groups contribute the same amount.

Second, a revenue maximizing charity should try to equalize $\tilde{X}_k(M)$ to the greatest extent possible. Within the pivotal minimum offer group $MRS_i(Y^*, y_i^*) = \frac{1}{M}$, assuming the group makes some positive contribution. Marginal rates of substitution are higher in the non-pivotal groups. Efficiency is therefore maximized when all groups are pivotal. It is easier to make different groups pivotal if the groups are more similar and it is often easier to keep groups similar in the face of uncertainty if the groups are larger, so there is benefit to larger groups.

Third, assuming that all groups can be made pivotal by equalizing $\tilde{X}_k(M)$, we have $\sum MRS_i(Y^*, y_i^*) = \frac{N}{M}$. This implies that more groups lead to higher efficiency if each group can be kept pivotal. If every player has their own group while remaining pivotal we have full efficiency, but this is only possible when players are identical.

There are therefore benefits to both small and large groups. Larger groups are more homogeneous and smaller groups lead to stronger incentive. A natural question is then, what is the optimal N? It seems natural that the answer could lie between $N = 1$ (the VCM) and $N = M$ (the homogeneous MOCM). Many existing charities use $N = 2$ when matching donations, but are they could be leaving money on the table by not considering other Ns.

4.1.1 Optimal N

We now consider the problem of an individual who wishes to pick N is order to maximize expected total contributions, as a charity might. We assume players are randomly assigned to groups of identical size. The key object in this analysis is $G(x, N, I)$ which gives the CDF for the group level contributions. Here I is the total number of people in the population.

The minimum group level contribution then has a CDF of $1 - (1 - G(x, N, I))^N$. The charity's problem then becomes.

$$\max_N N \int_0^\infty x d\left(1 - (1 - G(x, N, I))^N\right)$$

Using integration by parts we can transform this into a slightly simpler form

$$\max_N N \int_0^\infty (1 - G(x, N, I))^N dx$$

Increasing N here has several effects. There are two competing direct effects and two effects through $G(x, N, I)$. The first direct effect is that more groups means more groups to contribute. This gives the N multiplier out front. The second direct effect is the power of N which reflects the fact that ,ore groups means more opportunities for a lower minimum contribution level. The former effect tends to increase contribution levels while the latter tends to decrease them

There are also two competing effects of N on $G(\bullet)$. First higher N means fewer people in a group which generally drives down contributions, since there are fewer people to contribute. Second, more groups means a contribution will be matched more times, multiplying the marginal power of contributions. This increases the individual incentives to contribute. Which effects dominate depends on the details of the setup.

Empirically, it should be possible to estimate $G(x, N, I)$ for small Is using laboratory settings and to estimate for $N = 1$ or 2 and large I using existing charitable contribution data, but estimating for large I and $N \geq 3$ will almost certainly require specific assumption or data not currently or easily available. Still, we can explore the function space in order to get a better sense of what to expect.

To that end, we present two examples of different setups where we show how to calculate $G(x, N, I)$ and ultimately how to find the optimal N. These examples are chosen as extremes extremes with regards to crowding out. The first example has full crowding out with only one person contributing in each group. The second example has no crowding out with a person's contributions being independent other contributions within their group.

4.1.2 Baseline Example

First we consider something like the baseline utility function where people have a utility from the public good proportional to η_i as in equation 5. Assume that the δ_is are distributed identically based on CDF $F_\eta(\eta)$. The η_is are not required to sum to one.

The target contribution level for an individual is $g'^{-1}\left(\frac{1}{N\eta_i}\right)$, meaning that a person would contribute this amount if alone in their group. The N multiplier on η_i comes from the fact that there are $N-1$ other groups matching the player's donation. Therefore the target contribution levels are distributed $F_\eta\left(\frac{1}{Nu'(x)}\right)$. As discussed in Section 2.2, only the maximum target contribution level holds in equilibrium, so $G(x,N,I) = F_\eta\left(\frac{1}{Ng'(x)}\right)^{I/N}$.

Therefore the planner's objective becomes

$$\max_N N\int_0^\infty \left(1 - F_\eta\left(\frac{1}{Ng'(x)}\right)^{K/N}\right)^N dx$$

Here N shows up in 4 places corresponding to the four competing effects mentioned above.

For the sake of creating a complete example, we make several further assumptions. First we assume a simple G with two realizations of η_i: η_L and η_H with $\eta_L < \eta_H$. A player has η_L with a probability p_L. In this case the total contribution of the group only depends on the η_is through whether or not a η_H is present within the group. This means that the planner's objective becomes

$$\left(g'^{-1}\left(\frac{1}{N\eta_H}\right) - g'^{-1}\left(\frac{1}{N\eta_L}\right)\right)\left(1 - p_L^{I/N}\right)^N + g'^{-1}\left(\frac{1}{N\eta_L}\right)$$

Next assume $g(Y) = \ln(Y)$. The planner's objective simplifies further to

$$N\left(\eta_H - \eta_L\right)\left(1 - p_L^{I/N}\right)^N + N\eta_L$$

See Figure 1 to see how the population size influences optimal group size given different parameters.

As we can see, large populations generally lead to a larger optimal number of groups. However, this growth is slower when the percent of potential donors in the population is lower. Nonetheless, for realistic values of p_L and K, an optimal value for N above two seems reasonable.

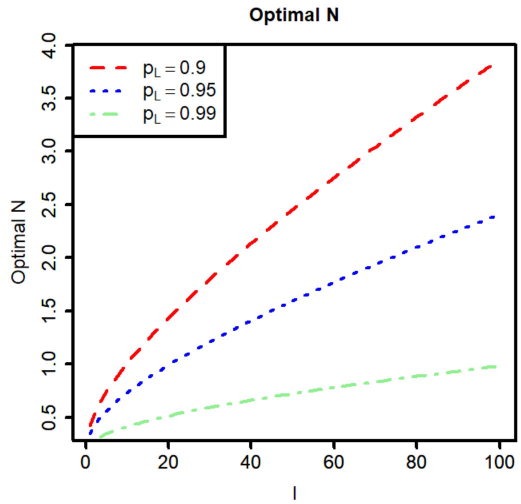

Figure 1: Optimal number of groups vs total population. $g(Y) = \ln(Y)$. Two player types: $\eta_H = 10$ and $\eta_L = 0$. p_L is probability of a player being low type. Note we do not restrict the number of groups to integers in order to better show the trend.

4.1.3 Alternative Specification

Next we consider an alternative specification with no crowding out at all. Assume

$$u_i(Y, \omega_i - y_i) = \eta_i Y - y_i^2$$

Again η_i is drawn independently from $F_\eta(\bullet)$ and does not have to sum to one. Here $y_i^* = \frac{N\eta_i}{2}$ which does not depend on the donations of others within the group. Assume $\frac{\eta_i}{2}$ has a mean μ and variance σ^2. Further assume that groups are large enough and μ is large relative to σ^2 such that a normal approximation can be used without issue for the zero lower bound.

In this case we get

$$G(x, N, I) = \Phi\left(x, I\mu, IN\sigma^2\right)$$

And the planner's objective function becomes

$$N \int_0^\infty \left(1 - \Phi\left(x, I\mu, IN\sigma^2\right)\right)^N dx$$

In numerical simulations the optimal group size in this setup is generally a single individual, so the lower bound on group size is primarily determined by how many people are needed for the normal approximation to be valid. When there is no crowding out, the smallest group size that still invokes the central limit theorem provides a lower limit on the optimal number of groups. Given some distributions of η_i it may be optimal to have an extremely large number of very small groups. This is particularly true when F_η is near degenerate.

5 Conclusion

In this paper we have presented a novel high-information/low-power public good mechanism as a potential alternative for scenarios where PPM may not perform well. Generally, high-information/low-power coercion mechanisms see substantial real world use and are of significant value when government intervention is unlikely or impossible (such as in the case of international agreements). We showed that the mechanism has nice properties including efficiency, fairness, equilibrium in dominant strategies, budget balance, and individual rationality. The fairness and dominant strategy properties provide advantages over the PPM.

We showed that, even in poor information conditions, the mechanism can perform well. In the presence manipulation, the mechanism should out-perform the VCM unless the problems are extreme. In Appendix C we show efficiency can be achieved if all players know each other's utilities even if the mechanism maker does not. We also discuss in Appendix D how players will generally be willing to provide accurate information about the utilities of others as long as it does not reveal information about themselveand that that players will be willing to report their own prices truthfully when public goods provision is particularly sensitive to price mismatches.

The biggest weakness for the MOCM is a combination of heterogeneity, imperfect information, and large groups. In this case we can modify the mechanism into the N-Group MOCM which takes advantage of the relative homogeneity of randomly chosen groups. The N-Group MOCM is a generalization of the standard matching donations mechanism commonly used by non-profits like National Public Radio or the Sierra Club to raise funds where one group of donors matches the donations from another group. Under some conditions the N-Group MOCM may open up more funding for charities than previous methods.

Overall, we contend that the MOCM is an excellent candidate for further research and potential use in several policy domains.

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A Proofs

A.1 Proof of Theorem 1

Take a some Lindahl Equilibrium (there may be multiple) with a contribution level Y_L^*. Note that 2 is a restricted version of

$$Y_i^* \in \arg\max_{Y_i} u_i(Y_i, \omega_i - p_i^* Y_i)$$

One solution of this is $Y_i^* = Y_L^*$. Note that, since $u_i(\bullet)$ is concave, the objective function is weakly increasing on the interval $[0, Y_L^*]$ and non-increasing beyond this interval.

By definition, the objective function is the same for all optimal choices of Y_i and is weakly worse for any $Y_i > Y_L^*$. Therefore, regardless of Y_{min}^i it is weakly optimal to pick $Y_i = Y_L^*$ and the player is indifferent between all elements of the set. $\square$

A.2 Proof of Proposition 1

To see this, note that 4 is the same as

$$Y_{inc}^i \in \arg\max_{Y_i} \tfrac{\delta_i}{w_i} * g\left(\min\left(Y_i, Y_{min}^i\right)\right) - \min\left(Y_i, Y_{min}^i\right)$$

Similar to the proof of Proposition 1, the concavity of $g(\bullet)$ guarantees that player i's objective function is increasing in $Y_i \forall Y_i \in [0, \sup(Y_{inc}^{i*}))$ where

$$Y_{inc}^{i*} = \arg\max_{Y_i} \left(\tfrac{\delta_i}{w_i} g\left(Y_i\right) - Y_i\right)$$

As such, it is a weakly dominant strategy to pick $Y_i \in Y_{inc}^{i*}$. If all players do this the minimum Y_i chosen gets implemented. From proposition 1, we know that the Y_{inc}^{i*}s are depend on $\frac{\delta_i}{w_i}$ in a manner that is monotone in the strong set order sense. Therefore, the implemented Y_i must be an element of the lowest Y_{inc}^{i*} corresponding with the lowest $\frac{\delta_i}{w_i}$. $\square$

A.3 Proof of Proposition 2

The manipulator will always want to pick $w_j \leq \delta_j$. We show this by contradiction. Note that he can implement Y^* by setting $w_j = \delta_j$. Say the manipulator picks $w_j > \delta_j$. Define

$$\tilde{Y_j} = \arg\max_{Y_i} \left(\delta_j * g\left(Y_j\right) - w_j Y_j\right)$$

We know

$$\delta_j * g\left(Y^*\right) - \delta_j Y^* \geq \delta_j * g\left(\tilde{Y_j}\right) - \delta_j \tilde{Y_j} > \delta_j * g\left(\tilde{Y_j}\right) - w_j \tilde{Y_j}$$

Hence w_j cannot be optimal.

Given his own mechanism weight $w_j > \delta_j$, the manipulator will want to maximize total contributions Y. To see this, note that

$$\tfrac{\delta_j}{w_j} * g\left(Y\right) - Y$$

is increasing in Y for all $Y \in [0, \bar{Y}_j^*]$ where

$$\bar{Y}_j^* \in \arg\max_{Y} \tfrac{\delta_j}{w_j} * g\left(Y\right) - Y$$

Note that $\bar{Y}_j^* > Y^*$ by Proposition 1. Therefore, the manipulator's objective function is increasing in Y for the entire range of Ys that is feasible given $p_j > \delta_j$.

Since the equilibrium Y depends on the lowest $\frac{\delta_i}{w_i}$, it is optimal to allocate the p_i in such a manner as equalizes this across all other individuals.

We know

$$1 - w_j = \sum_{i \neq j} w_i$$

So we have

$$w_i = \tfrac{1-p_j}{1-\delta_j} \delta_i$$

Which satisfies the summand and the requirement that $\frac{w_j}{\delta_j}$ be equalized.

Therefore,

$$\frac{\delta_i}{w_i} = \frac{1-\delta_j}{1-w_j} \forall i \neq j$$

meaning all players other than j will make offers that implement $Y \in Y^*_{man}$, and the manipulating player j will make an offer weakly greater than Y^*_{man}.

$\square$

A.4 Proof of Corollary 1

Since $w_j \in [0, \delta_j]$, we know that $\frac{1-\delta_j}{1-w_j} \in [1-\delta_j, 1]$. This implies that manipulation has a higher κ as long as $\max_i (\delta_i) \leq 1 - \delta_j$ where j is the manipulating agent.

It is only possible for the manipulation mechanisms κ to be lower if $\max_i (\delta_i) > 1 - \delta_j$. Note

$$1 - \delta_j = \sum_{i \neq j} \delta_i \geq \delta_i \forall i \neq j$$

So this can only happen if $\delta_j > 1/2$. $\square$

A.5 Proof of Proposition 3

Say that $k^* = \arg\min_k \tilde{X}_k(M)$ is the index of the pivotal group.

In equilibrium it must be that every group is providing the same total offers. Due to the refund mechanism, money beyond the minimum group offer gets redistributed among all group members evenly. This means that if a group is offering more than the minimum group offer any member can reduce their offer and increase their payoff. As such we can assume that all players are restricted to be unable to make offers which could increase their group offer beyond the minimum group offer from other groups WLOG (minimum restriction 0).

First consider group k^*. They are essentially playing the VCM with a multiplier on benefits and a restriction that is non-binding at the equilibrium corresponding to $\tilde{X}_{k^*}(M)$. Hence, they have no reason to deviate from that offer level.

Next consider group $k \neq k^*$. They are essentially playing the VCM with a multiplier on benefits and a restriction that $\sum_{i \in S_k} x_i \leq \tilde{X}_{k^*}(M)$. Due to the concave nature of utilities and by construction of $\tilde{X}_k(M)$, the restriction will be binding. If total contributions are less than $\tilde{X}_k(M)$ at least one player will be able to profitably deviate by increasing offer by definition. $\square$

B Manipulation Worse than VCM Example

Consider the following setup. There are two individuals with $\delta_1 = \frac{3}{4}$ and $\delta_2 = \frac{1}{4}$. Assume $g(y)$ is piece-wise linear and weakly concave. The first section has a horizontal length Y_1 and has a slope $4 + \epsilon$. The second piece has a horizontal length Y_2 and has slope $\frac{4}{3} + \epsilon$. Past the second section, $g(y)$has a slope of zero. The example works with strictly concave approximations, but it is easier to see in the piece-wise linear case.

If player 1 is the manipulator, he can implement $Y = Y_1$ choosing $w_1 = 0$ because that is what Player 2 will naturally bid. He can also implement $Y = Y_1 + Y_2$ by choosing w_1 such that the Player 2 will choose the higher level. This requires the following condition

$$\tfrac{1}{4}(4/3 + \epsilon) \geq w_2$$

Since weights must sum to one, this can be rewritten as

$$w_1 \geq 2/3 - \epsilon/4$$

We can make Y_2 arbitrarily small such that the utility gains going from the first kink to the second are negligible, but the difference in cost is approximately $(2/3)Y_1$. If Y_1 is relatively large, manipulator will implement Y_1 which is slightly smaller (and slightly less efficient) than the natural mechanism outcome.

C Checked Leader MOCM

The MOCM as described requires the mechanism maker to have full knowledge of the preferences of all individuals involved in the public good as is typical in discussions of the PPM. However, there are other informational environments that the PPM can be adapted to. For example, the PQ mechanism of Van Essen and Walker (2017) works well in environments where the mechanism maker has no knowledge but preferences are common knowledge among participants.

We can also adapt the MOCM for such an environment. There are actually a number of ways to do this, but we will focus on the most illustrative called Checked Leader MOCM (CL-MOCM).

This mechanism enriches the MOCM into a 3 stage game. There are two players (a Leader and a Checker) who each have a special role in this mechanism. They should be selected based on the amount of information they possess with the leader requiring information about the other participants and the checker requiring information about the leader. Say that Player 1 is the Checker and Player 2 is the Leader

The Checked Leader MOCM goes like this

1. The Checker picks w_2 for the Leader
2. The Leader picks $\boldsymbol{w}_{-2}$ (all w_i other than w_2) subject to $\sum_{i \neq 2} w_i = 1 - w_2$
3. Players play MOCM game

After the final stage, the checker faces a penalty if $Y_2 \neq Y_3$. This guarantees w_2 is picked correctly, since any other pick will cause discrepancy. Note that this penalty does remove the guarantee of budget balance and of individual rationality for the checker, although this will not be relevant in equilibrium and the mechanism can be guaranteed not to run a deficit.

In this setup we have the following result

Proposition 4. *Say that utilities take the form in expression 3 and are increasing, strictly concave, and twice continuously differentiable. Assuming that the leader knows all preferences within the group and that the checker knows the preference of the leader, then there exists a SPE of the CL-MOCM which implements a Lindahl Equilibrium and reveals Lindahl Prices.*

Proof.

Stage 3: We work by backwards induction. By strict concavity, we know in the final stage that each person single weakly dominant strategy that is decreasing in w_i in the strong set order sense. Call the resulting adjusted offers $Y_i(w_i)$.

Stage 2: Player 2's selection of $\boldsymbol{w}_{-2}$ only influences his payoff through the restriction it places on his maximum effective adjusted offer. This restriction is $\bar{Y} = \min_{i \neq 2} Y_i(w_i)$. By monotonicity and strict concavity and continuous differentiability, this restriction is maximized when $Y_i(w_i) = Y_j(w_j) \forall i, j \neq 2$. Therefore Player 2 will always choose $\boldsymbol{w}_{-2}$ to equalize adjusted offers.

Stage 1: Player 1 may have some incentive to set w_2 higher than p_2^* ordinarily (in order to slightly reduce w_1) but this incentive can be counteracted by the arbitrarily large mismatch penalty. To avoid this penalty, Player 1 must set $w_2$such that Player 2 will choose the same Y as Player 3. In stage 2 we showed that this is also the same Y chosen by all players other than Player 2. This means that the resulting weights meet the definition of Lindahl Prices.

We can see that if Player 1 picks $w_2 = p_2^*$ then Player 2 will pick $\boldsymbol{w}_{-2} = \boldsymbol{p}_{-2}^*$ and everyone will make adjusted offers of Y_L^*

□

Note this proposition used the general utility form, not the quasi linear form.

This proposition takes advantage of two facts: first that players wish to reveal their information about the preferences of others; second, that when prices are correct, adjusted contributions match. This is not the only mechanism which can be used to elicit Lindahl Prices when they are common knowledge among participants. For example, one could also use a mechanism where each player proposes a price vector and then the true price vector is equal to the most frequently proposed vector. If no two players agree, then the MOCM is not executed. This mechanism maintains the guaranteed budget balance and individual rationality, but it pays a cost in a much greater multiplicity of equilibria.

D Information Extraction Example

D.1 Other's Price Revelation

Players can be induced to reveal information about others if it does not reveal information about their own price.

Consider a proportional benefit environment where each player knows their own δ_i where $\delta_i \in \{\delta^L, \delta^M, \delta^H\}$ where $\delta^L < \delta^M < \delta^H$. Only one player can have each δ. A priori each matching of δ to player is equally likely.

Say that each player receives $s_i \in \{1,2,3\}\backslash\{i\}$ which is equal to j if $\delta_j > \delta_k$ with probability $\rho > 0.5$. In other words it is an informative signal about which other player has a higher δ.

Each player sends the mechanism maker a signal $s_i \in \{1,2,3\}\backslash\{i\}$. The mechanism maker then selects a vector of prices $\boldsymbol{p}$ summing to 1 as a function of messaged received. Consider a mechanism which assigns $w_i = \delta^H$ for whichever i corresponds to the most signals received and $w_j = \delta^L$ for whichever j corresponds to the fewest received signals. The remaining player receives a price δ^M. Ties are broken uniformly randomly. Note that this is not necessarily the optimal mechanism in terms of efficiency.

Call this the other price information extraction game.

Remark 2. In the information extraction game, each player revealing their signal is a Subgame Perfect Equilibrium.

Proof. A player's expected payoff is decreasing in their price and increasing in the minimum adjusted offer of other players. Given two players and two prices, the minimum adjusted offer between those players is higher if the player with the higher δ receives the higher price.

Given honest revelation by all players, the distribution of a player i's prices is independent of their signal sent given their lack of knowledge about other player's signals. Further, it can be shown by checking cases that the given a price w_i the probability of other players receiving their more appropriate price matching is strictly higher when player i reveals truthfully. Therefore revealing truthfully is optimal in equilibrium. □

D.2 Own Price Revelation

When the efficiency motive dominates and the incentive for manipulation is low, it can be possible to get players to truthfully reveal their own prices.

Again consider a proportional benefit environment where each player knows their own δ_i where $\delta_i \in \{\delta^L, \delta^M, \delta^H\}$ where $\delta^L < \delta^M < \delta^H$. Only one player can have each δ. A priori each matching of δ to player is equally likely. We denote players in this by L, M,or H depending on their δ. This time we assume a provision point-like utility function

$$g(Y) = \begin{cases} Y(1+\epsilon) & Y < 10 \\ 10 + 10\epsilon & Y \geq 10 \end{cases}$$

Call this the matching prices environment.

Note under this public good production technology, if k is large enough, that guarantees that the mechanism runner cannot guess the order of δ wrong and still produce a positive value from the public good. We show this with the following Lemma.

Lemma 1. *We can guarantee that $Y = 0$ in the matching prices environment whenever $w_i = \delta^i$ is assigned to a player with $\delta^{j \neq i}$ if $\delta^M(1+\epsilon) < \delta^H$ and $\delta^L(1+\epsilon) < \delta^M$*

Proof.

One of these will occur in any mis-atribution of weights

Case 1 Setting $w_M = \delta_H$.

Minimum Y_i will be player M's. His unconstrained optimization becomes

$$\delta^M Y(1+\epsilon) - \delta^H Y$$

So $Y_M = 0$ if

$$\delta^M(1+\epsilon) < \delta^H$$

Case 2 Setting $w_L = \delta_M$.

Minimum Y_i will be player L's. His unconstrained optimization becomes

$$\delta^L Y(1+\epsilon) - \delta^M Y$$

So $Y_L = 0$ if

$$\delta^L(1+\epsilon) < \delta^M$$

Case 3 Setting $w_L = \delta_H$.

Minimum Y_i will be player L's. His unconstrained optimization becomes

$$\delta^L Y(1+\epsilon) - \delta^H Y$$

So $Y_L = 0$ if

$$\delta^L(1+\epsilon) < \delta^H$$

Note the case 3 condition is implied by the conditions from the other conditions

For the remaining discussion assume δs satisfy the conditions. Each player sends the mechanism maker a signal $s_i \in \{L, M, H\}$. The mechanism maker then attempts to match the correct weights to each player.

We will assume that the credulous mechanism works as follows. If a player is the only one to send signal $s_i = j$ then they receive $w_i = \delta^j$. If multiple players send the same signal, then one is chosen randomly to receive the corresponding price and the other receives the remaining price. If all players send the same signal, then all receive w_i equal to a random δ^j. We call the game created by using the credulous mechanism in the matching prices environment the own price information extraction game.

We can now present the following corollary

Corollary 2. *There is an equilibrium in the own price information extraction game in weakly dominant strategies where each player reveals δ_i truthfully.*

Proof. We know that the player gets one of two payoffs: (1) the $\delta_i 10\epsilon$ they get when each player gets the correct price and (2) the 0 that they get when at least one player gets the wrong price . We also know (1) > (2). Therefore, the best action is the one which maximizes the chance of (1).

Case 1: All other players reveal truthfully. Player i revealing truthfully generates (1) with certainty.

Case 2: One other player sends an incorrect signal s_j. Sending the correct s_i generates (1) with probability 1/2 regardless of which wrong signal was sent. Sending a wrong signal not equal to s_j guarantees (2) since there is now a wrong signal uncontested by a right signal. Sending a wrong signal equal to s_j will lead to all players sending the same signal which means (1) is generated with a probability of 1/6.

Case 3: Both other players send an incorrect signal. There will always be an uncontested incorrect signal, so (2) is guaranteed.

Therefore, reporting truthfully is a weakly dominant strategy. $\square$